TRINITY COLLEGE

Year 12 Mathematics Specialist 3,4
Test 1 2021

Section 1 Calculator Free

Complex Numbers, Functions and Sketching Graphs

STUDENT'S NAME _____

DATE: Wednesday 3 March **TIME:** 20 minutes **MARKS**: 20

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

Given $z = 1 + \sqrt{3}i$, determine

(a) $3zi$ [2]

(b) $Arg\left(\frac{-4}{z}\right)$ [2]

2. (5 marks)

Consider $f(z) = z^3 - 4z^2 + 6z - 4$ where z is a complex number.

(a) Show that $(z-2)$ is a factor of $f(z)$. [2]

(b) Solve the equation $z^3 - 4z^2 + 6z - 4 = 0$ [3]

3. (6 marks)

For the equation $z^4 = -2i$;

(a) Solve the equation giving the solutions in polar form. [4]

(b) A regular polygon is formed from the roots to the equation. Determine the exact area of the polygon. [2]

4. (5 marks)

Functions f and g are defined as $f(x) = x^2 - 1$ and $g(x) = \frac{1}{\sqrt{x}}$

(a) Determine an expression for $g \circ f(x)$. [1]

(b) For $g \circ f(x)$, state the:

(i) domain. [2]

(ii) range. [2]

TRINITY COLLEGE

Year 12 Mathematics Specialist 3,4
Test 1 2021

Section 2 Calculator Assumed

Complex Numbers, Functions and Sketching Graphs

STUDENT'S NAME _____

DATE: Wednesday 3 March **TIME:** 30 minutes **MARKS**: 30

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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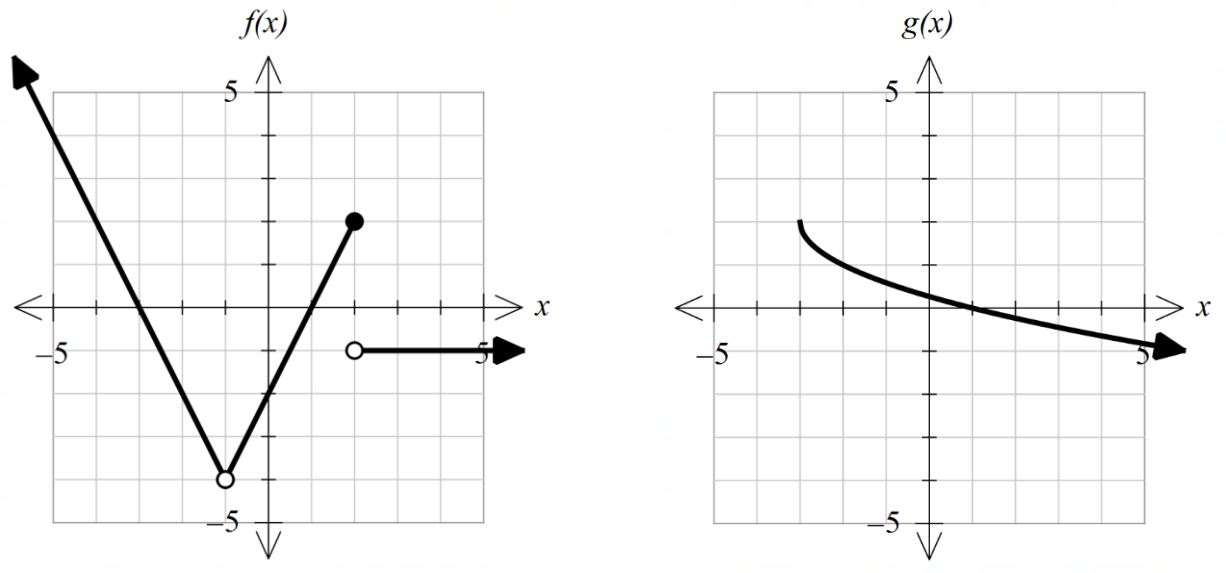
5. (12 marks)

The sketch of the graph of $y = f(x)$ and $y = g(x)$ is shown below:



(a) Sketch the graph of $y = g^{-1}(x)$ on the axes above. [2]

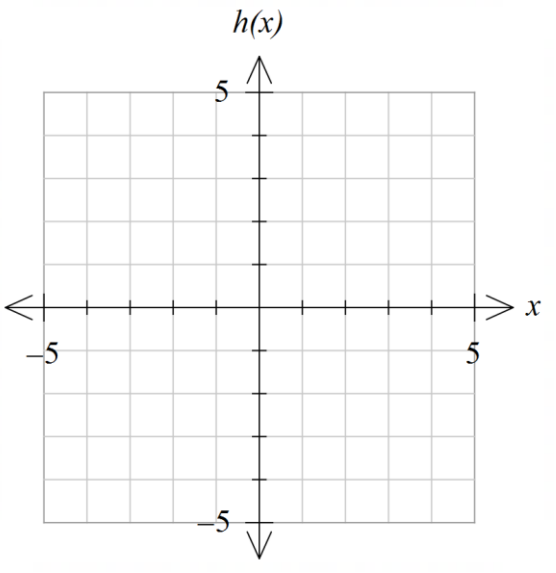
(b) Calculate the value of:

(i) $f \circ g(-3)$ [1]

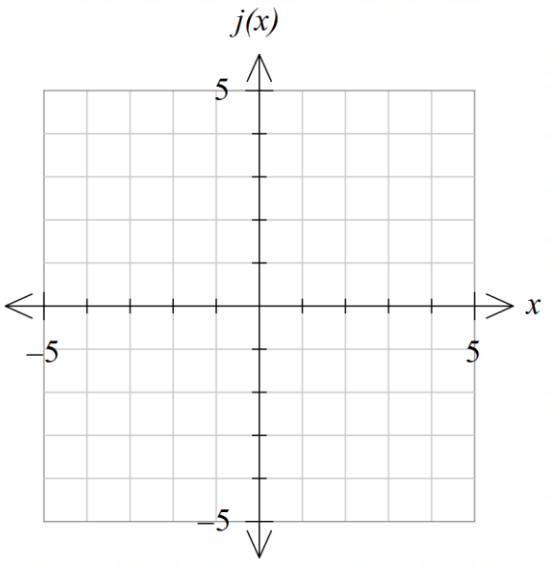
(ii) $f \circ g^{-1}(2)$ [1]

(iii) Explain why it is not possible to calculate $g \circ f^{-1}(2)$ [1]

(c) Sketch the graph of $h(x) = \dfrac{1}{f(x)}$ on the axes below. [4]

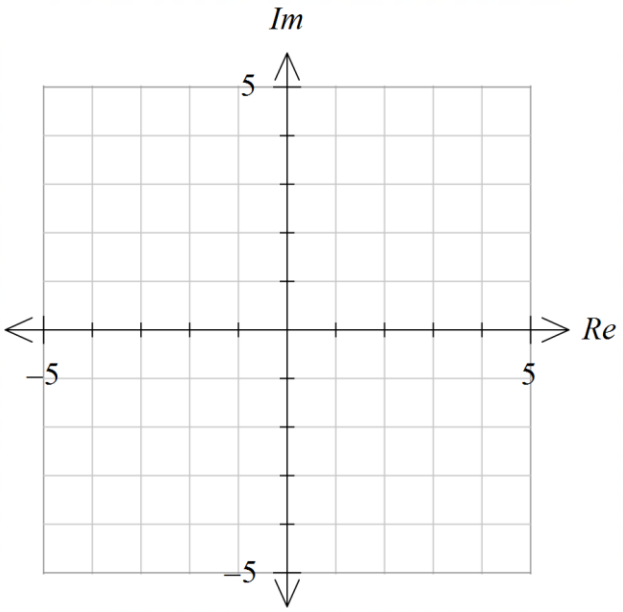


(d) Sketch the graph of $j(x) = \left| f\left|x\right| \right|$ [3]

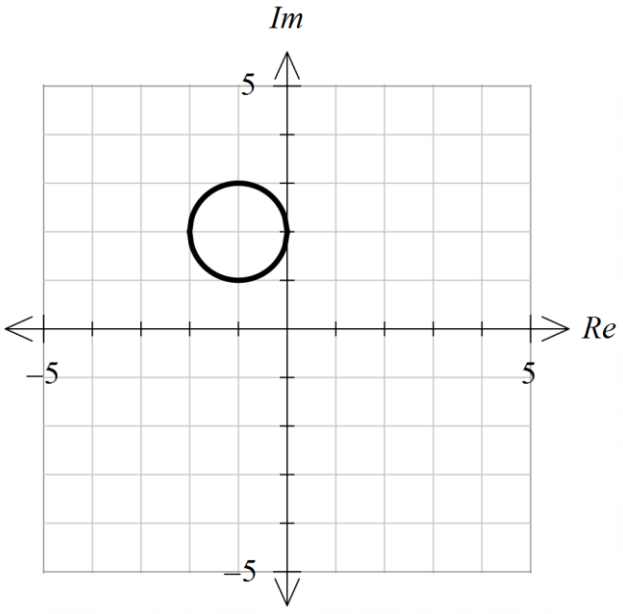


6. (8 marks)

(a) Sketch the locus of the equation $|z+2| \geq |z-i|$ in the Argand diagram below. [3]



(b) The sketch of the locus of a complex number $z = x + iy$ is shown below.

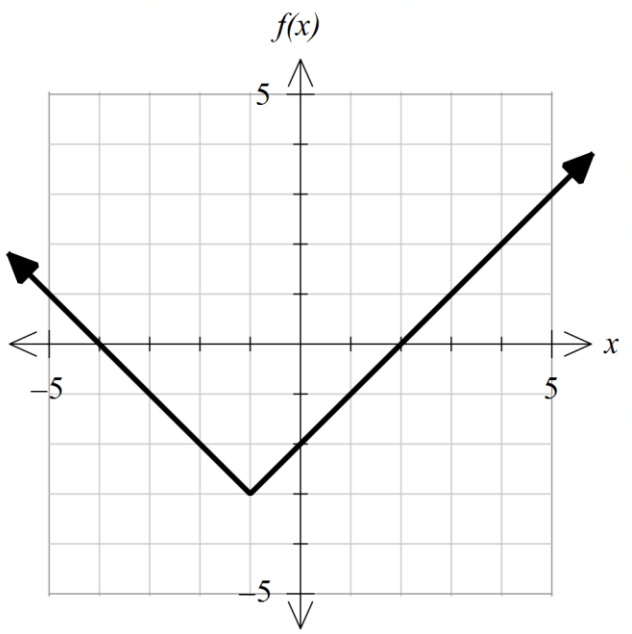


(i) Given that the equation for the above locus is written as $|z+a| = b$, determine the value of a and b. [2]

(ii) Determine the minimum value for $Arg(z+1)$ as an exact value. [3]

7. (4 marks)

The sketch of the graph of $y = f(x)$ is shown below.



Consider the equation $|f(x)| = k$ where k is any real constant.

Determine the value(s) of k such that $|f(x)| = k$ has two real solutions.

8. (6 marks)

Sketch the locus of points in the case where z satisfies $\{z; z \in \mathbb{C}, Arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}\}$